## Treatment of the topic

Whatever the title of the EE, students must apply good mathematical practice that is relevant to the chosen topic, including:

- data analysed using appropriate techniques
- arguments correctly reasoned
- situations modelled using correct methodology
- problems clearly stated and techniques at the correct level of sophistication applied to their solution.

## Research methods

Students must be advised that mathematical research is a long-term and open-ended exploration of a set of related mathematical problems that are based on personal observations. The answers to these problems connect to and build upon each other over time.

Students' research should be guided by analysis of primary and secondary sources.

A primary source for research in mathematics involves:

- data-gathering
- visualization
- abstraction
- conjecturing
- proof.

A secondary source of research refers to a comprehensive review of scholarly work, including books, journal articles or essays in an edited collection.

A literature review for mathematics might not be as extensive as in other subjects, but students are expected to demonstrate their knowledge and understanding of the mathematics they are using in the context of the broader discipline, for example how the mathematics they are using has been applied before, or in a different area to the one they are investigating.

### Writing the essay

Throughout the EE students should communicate mathematically:

- describing their way of thinking
- writing definitions and conjectures
- using symbols, theorems, graphs and diagrams
- justifying their conclusions.

There must be sufficient explanation and commentary throughout the essay to ensure that the reader does not lose sight of its purpose in a mass of mathematical symbols, formulas and analysis.



The unique disciplines of mathematics must be respected throughout. Relevant graphs and diagrams are often important and should be incorporated in the body of the essay, not relegated to an appendix. However, lengthy printouts, tables of results and computer programs should not be allowed to interrupt the development of the essay, and should appear separately as footnotes or in an appendix. Proofs of key results may be included, but proofs of standard results should be either omitted or, if they illustrate an important point, included in an appendix.

# Examples of topics, research questions and suggested approaches

Once students have identified their topic and written their research question, they can decide how to research their answer. They may find it helpful to write a statement outlining their broad approach. These examples are for guidance only.

Торіс	The geometry of navigation
Research question	What was the role of mathematics, and geometry in particular, in navigation when we relied on the stars? Does it still play a part now we have man-made satellites?
Approach	Using one of the two geometric representations of the Earth (spherical or ellipsoidal), describe how maps and charts were produced to assist navigators in the past.

Торіс	Square-triangular numbers and Pell's equation
Research question	How many square numbers are also triangular numbers, where are they and what other problems lead to Pell's equation?
Approach	A description of square and triangular numbers, and how the locations of numbers that are both are solutions of Pell's equation. Some other problems, perhaps in number theory and geometry, that lead to the equation could be described, with a brief history of the equation included.

Торіс	The exponential function and the measurement of age and growth
Research question	How does the exponential function, and its calculus, inform areas of science such as nuclear physics, geology, anthropology or demography?
Approach	Use one of the settings where exponential growth applies, perhaps modelling the world's population, to describe the phenomenon. Show how it is applicable in mathematical models of other real situations.

Торіс	Approximation of irrational numbers by rational numbers
Research question	How well can $\pi$ , $e$ , $\sqrt{2}$ and other irrationals be approximated by rational numbers?
Approach	Use the decimal representation of irrational numbers as a starting point to introduce approximation by rationals. Show how a continued fraction expansion of an irrational can also provide rational approximation, and discuss error bounds and orders of approximation.



Торіс	Archimedes' calculation of areas
Research question	What is the legacy of Archimedes' calculations of circular and parabolic areas in today's methods of integration?
Approach	Describe how Archimedes determined the area of a circle by using inscribed polygons, leading also to his measurement of $\pi$ . Continue with a description of his method of discovery for calculating the area of a parabola.

### An important note on "double-dipping"

Students must ensure that their EE does not duplicate other work they are submitting for the Diploma Programme. For example, students are not permitted to repeat any of the mathematics in their IA in their EE, or vice versa.

#### The mathematics EE and internal assessment

An EE in mathematics is not an extension of the internal assessment (IA) task. Students must ensure that they understand the differences between the two.

- The EE is a more substantial piece of work that requires formal research.
- The IA is an exploration of an idea in mathematics.

It is not appropriate for a student to choose the same topic for an EE as the IA. There would be too much danger of duplication and it must therefore be discouraged.

Supervisors play an important role in guiding students on these distinctions. Students risk their diploma if academic misconduct is detected.

